Normal and Superconducting state of doped Strontium Titanate

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University of Minnesota

Collaborators

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R. M. Fernandes

C. Leighton

E. McCalla
[*McGill]

Outline

- The Normal state
  - Weakly correlated Fermi liquid
  - Unusual mechanism for T^2 resistivity?

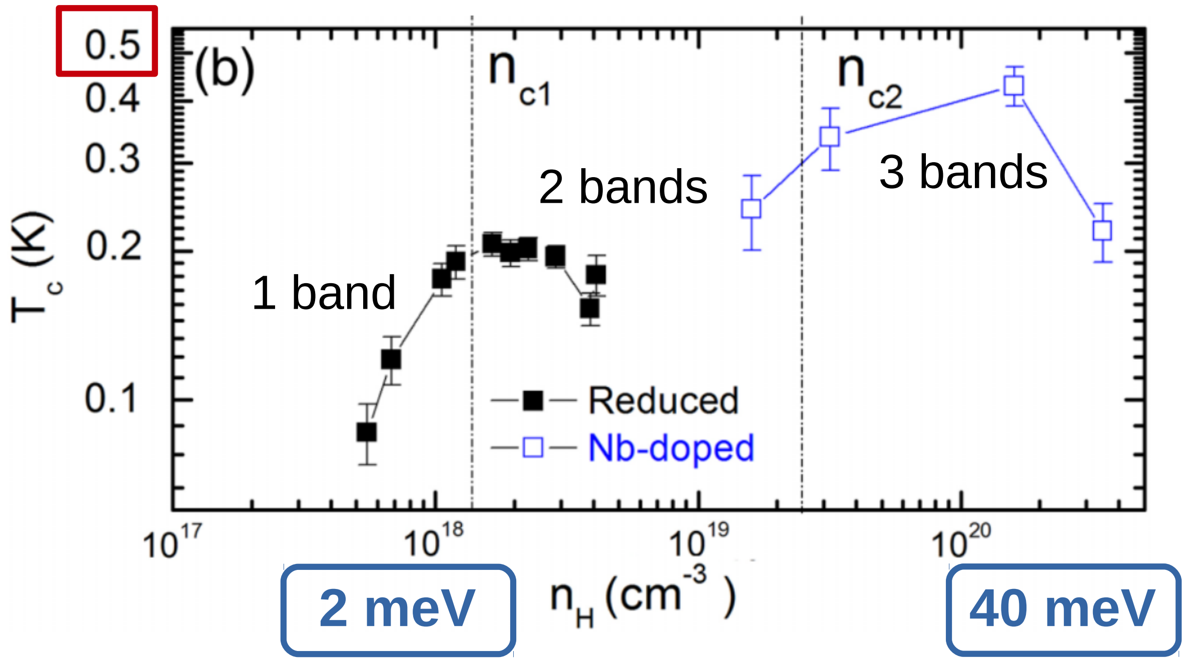
- Superconducting state
  - Electron-phonon mechanism.

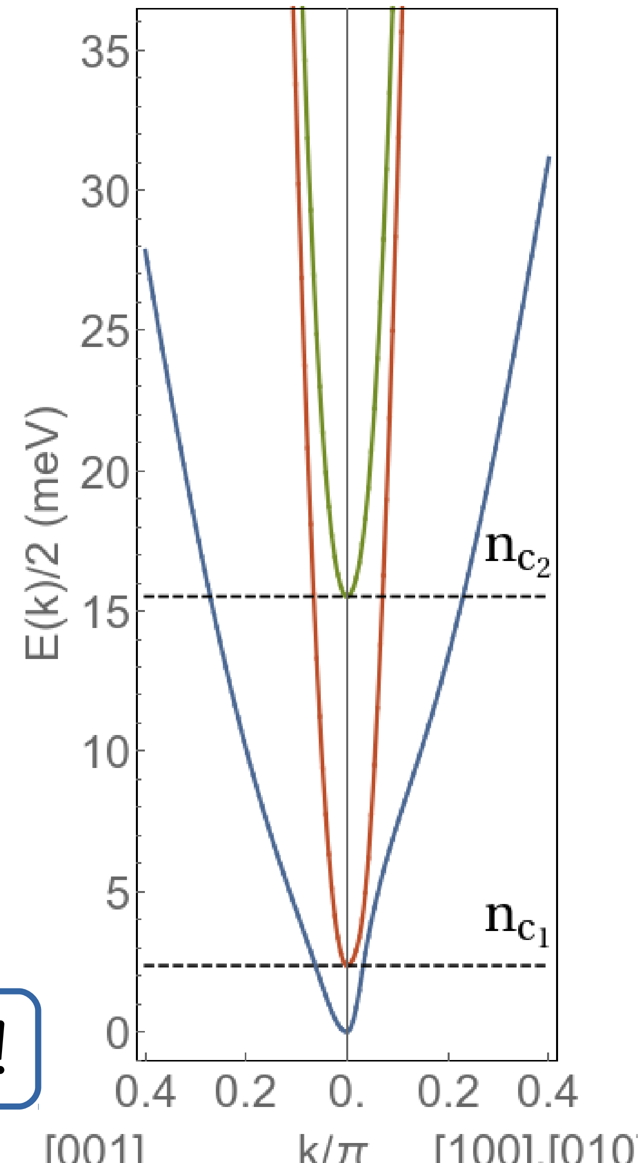
    Non-BCS behavior in the limit of small Fermi energy

Brief introduction & Puzzles

Schooley, Hosler, Cohen, PRL (64)

Lin et al PRL (14)





Low ε_F!!

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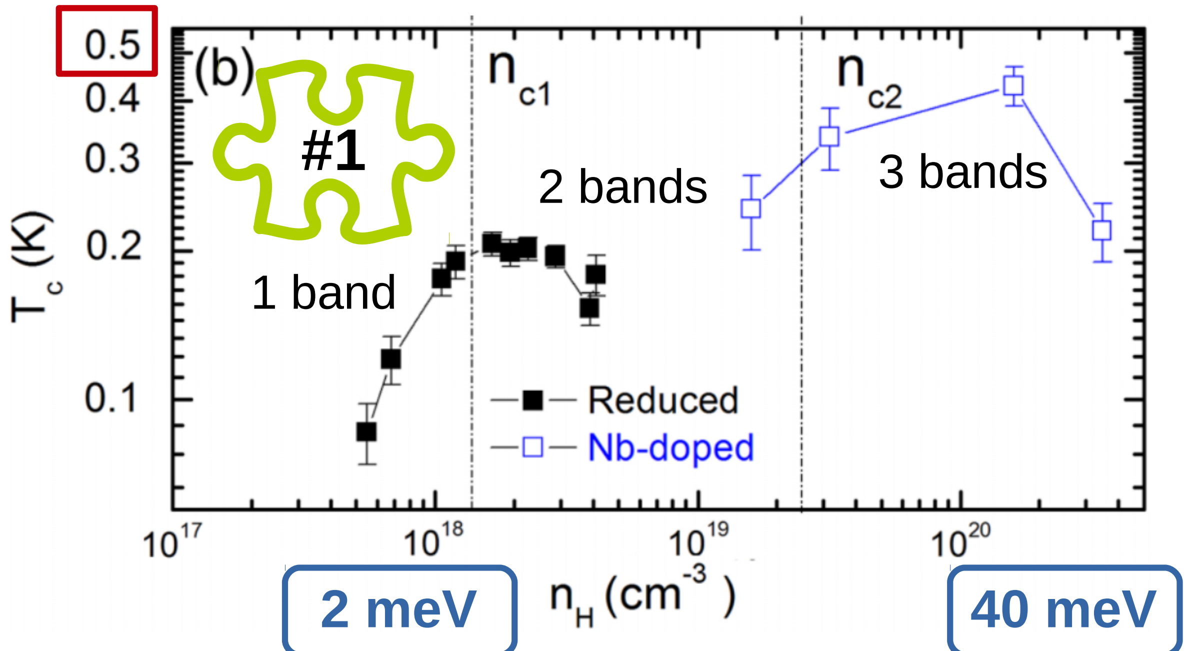

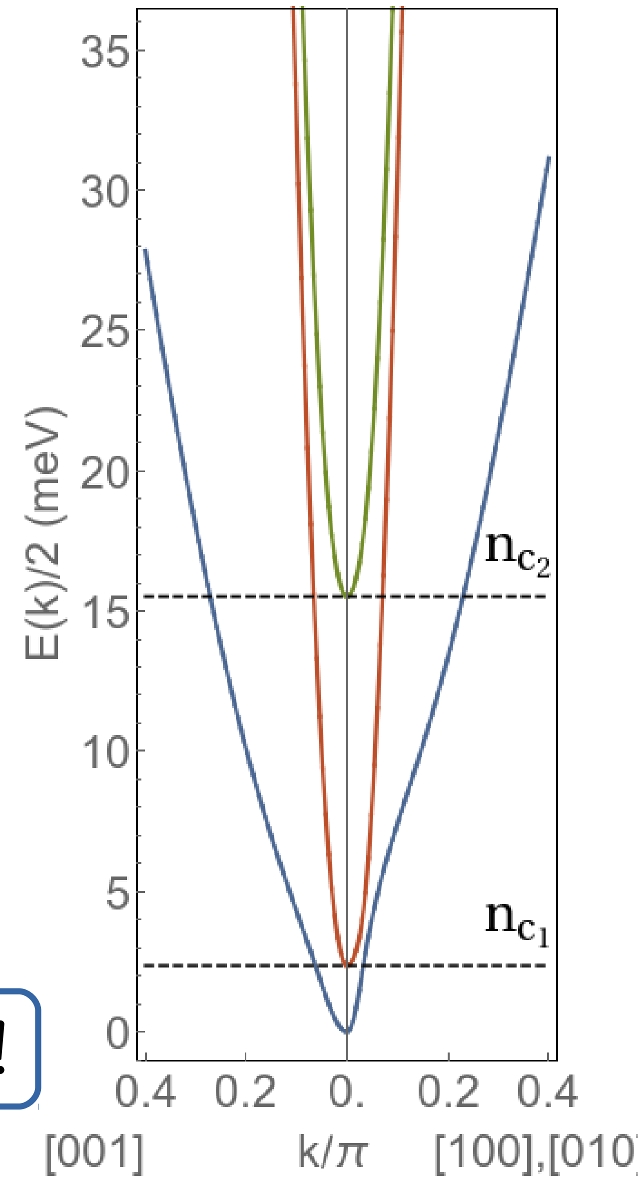

Nature of the correlated electron state?
Mechanism for T-square resistivity e-e scattering?

$$\rho = \rho_0 + AT^2$$

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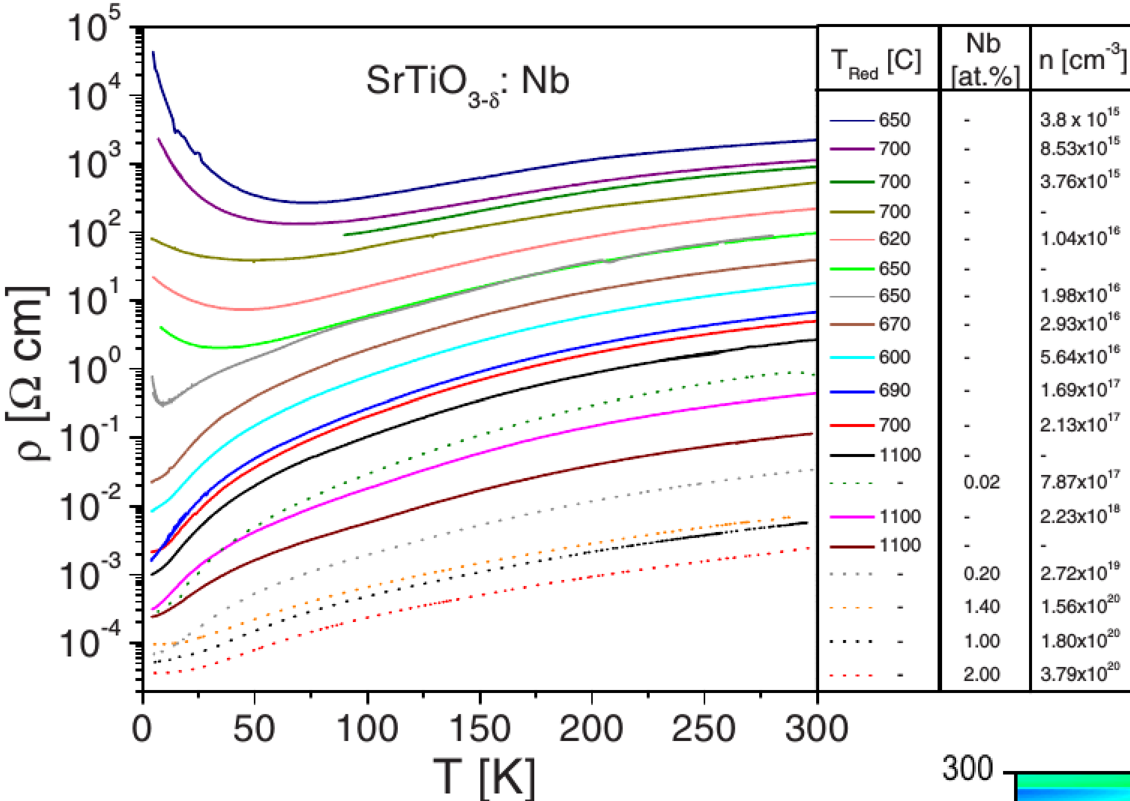
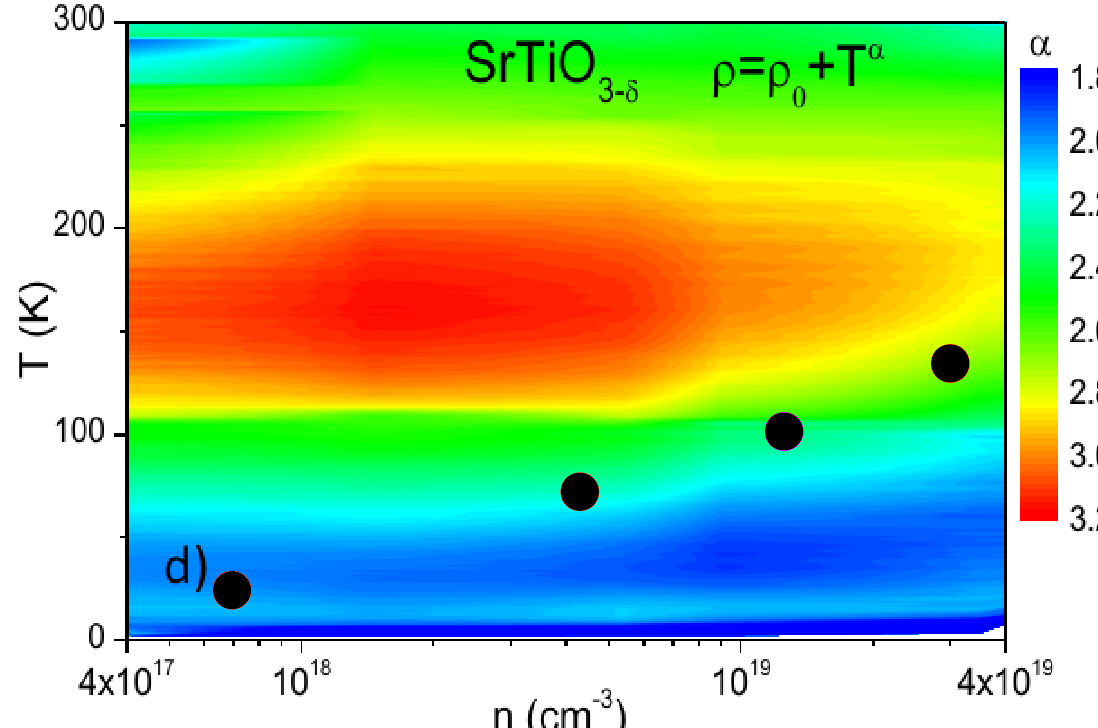


Spinelli et al PRB (10)



● = T_F

Collignon et al arXiv (18)

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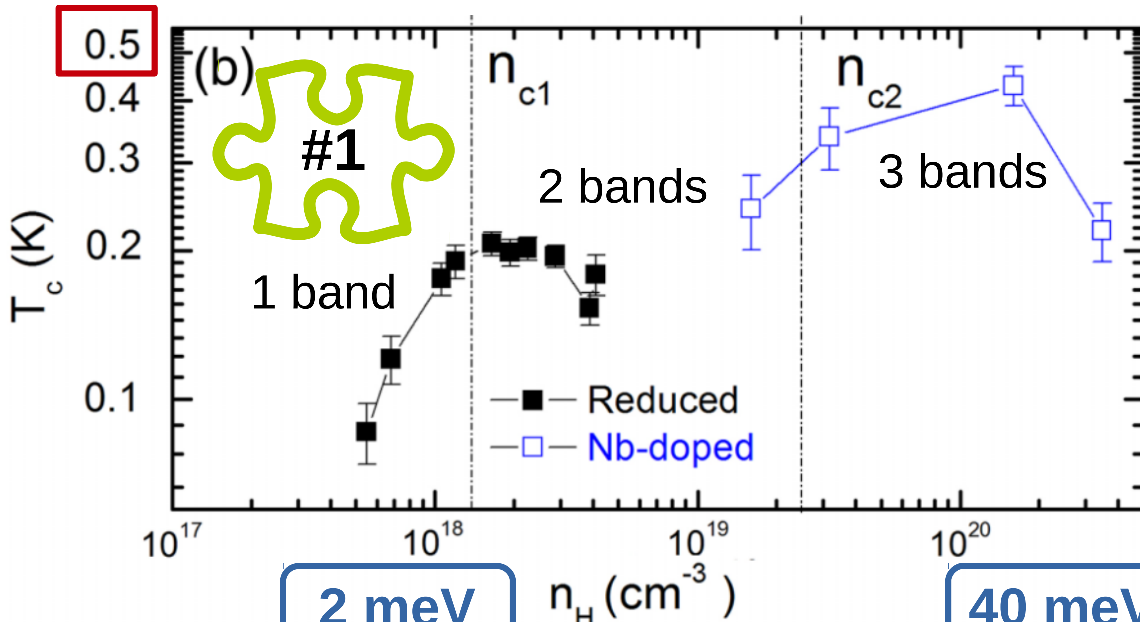

2 meV **40 meV**

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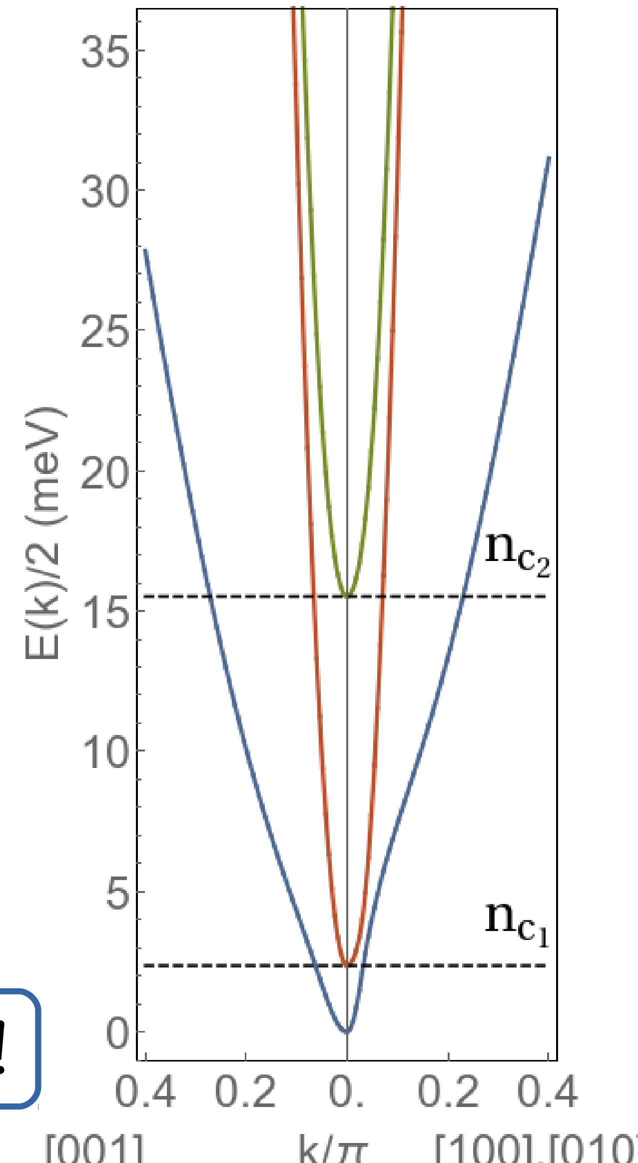

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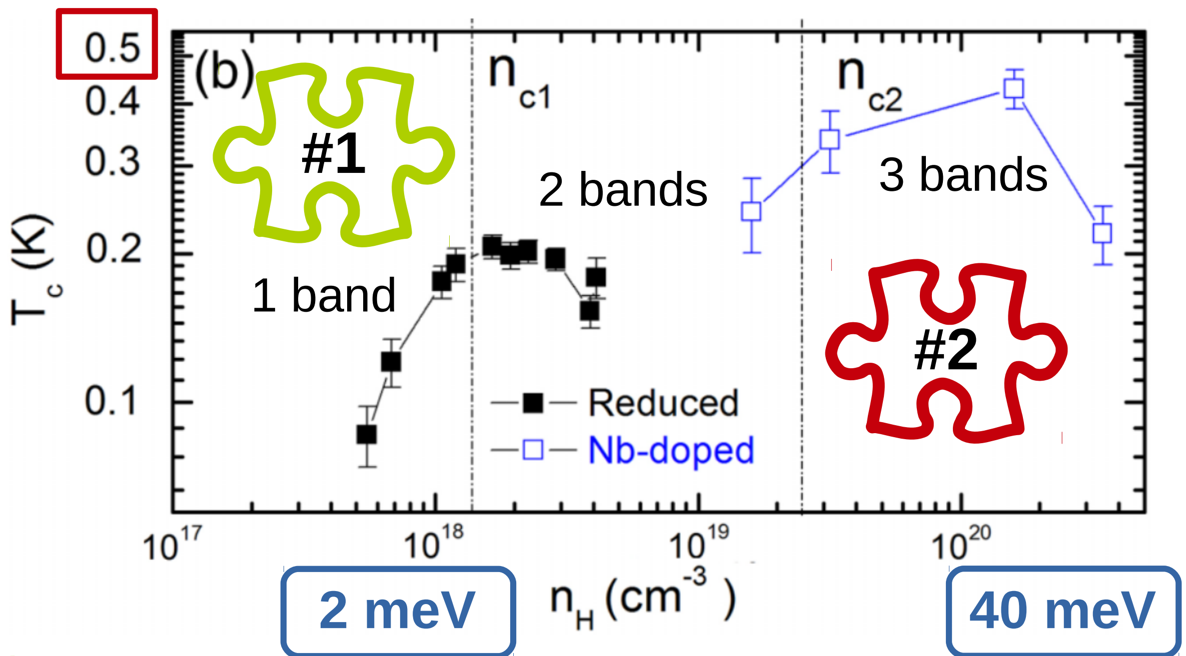

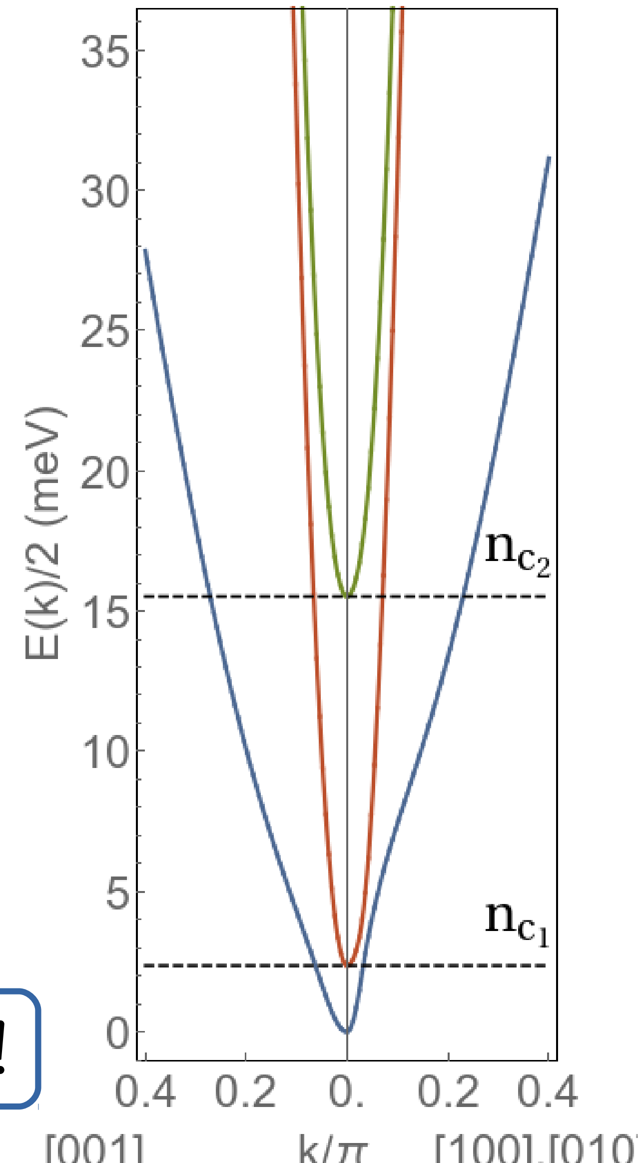

#1

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#2

Mechanism for SC?

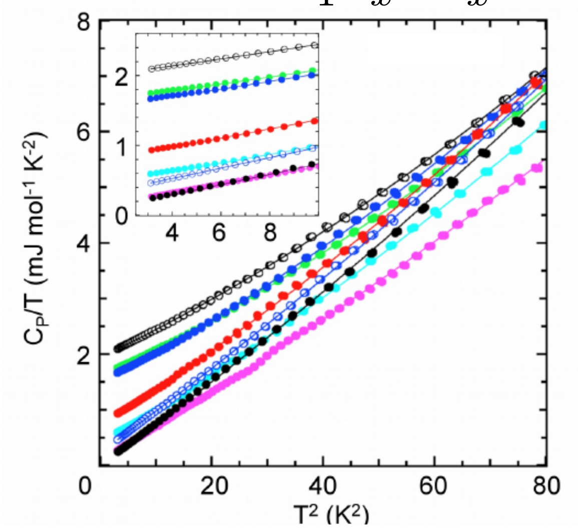
$\Omega_{LO} >> \varepsilon_F$

Low ε_F!!

Normal state

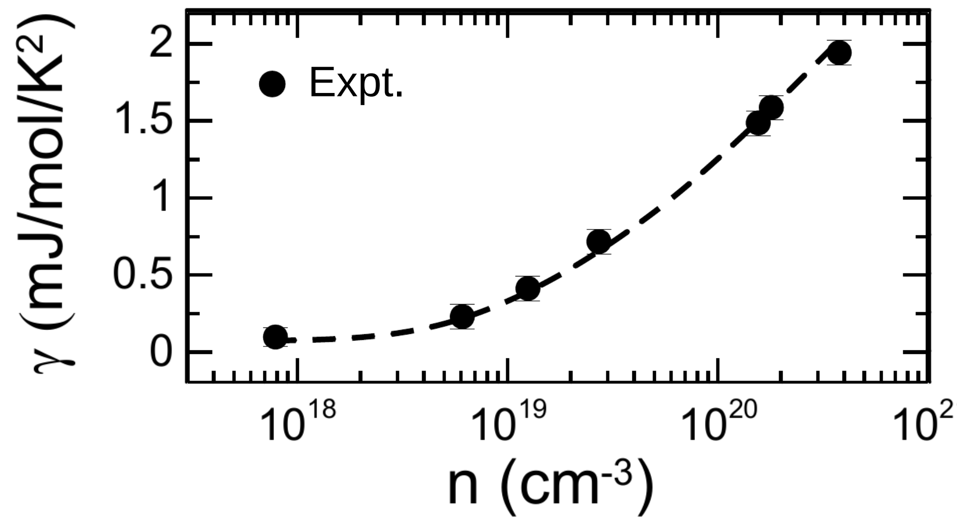
Weakly correlated Fermi liquid

Experiment $\mathrm{Sr}_{1-x}\mathrm{Nb}_x\mathrm{TiO}_3$ single crystals



Contribution of electrons to the specific heat:

$$\frac{C}{T} = \gamma + \ldots$$



McCalla, MNG, Cassuto, Fernandes, Leighton, in prep.

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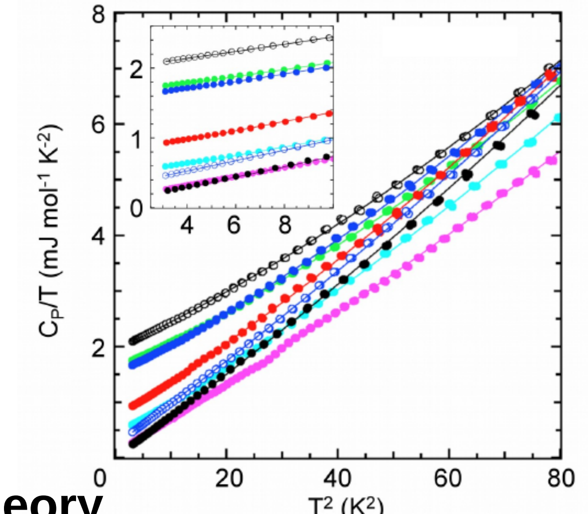

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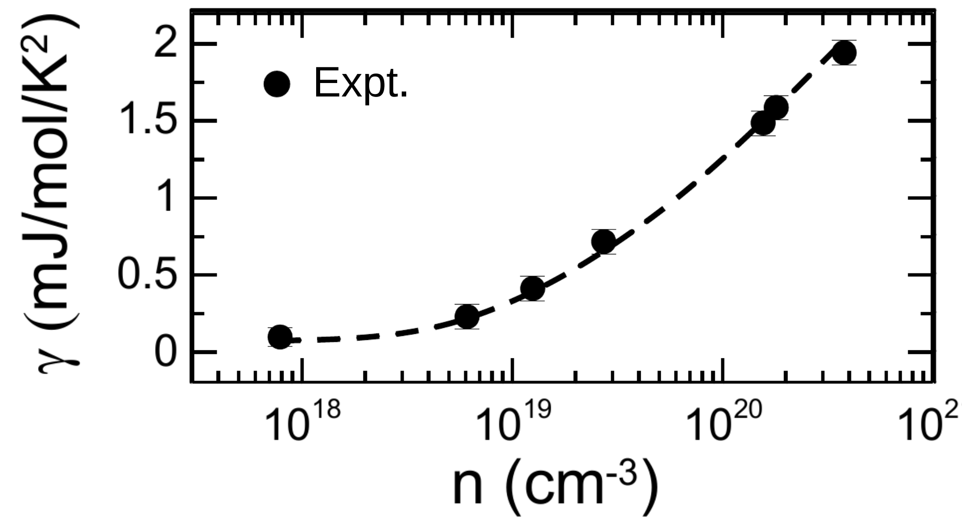
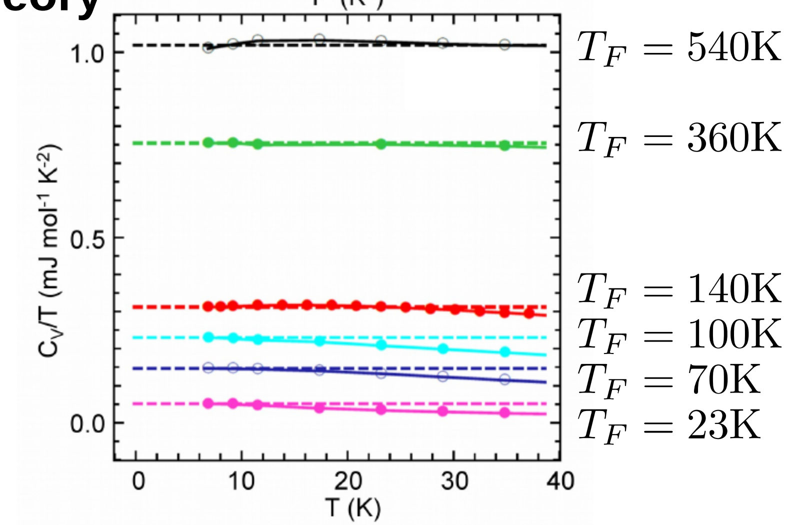
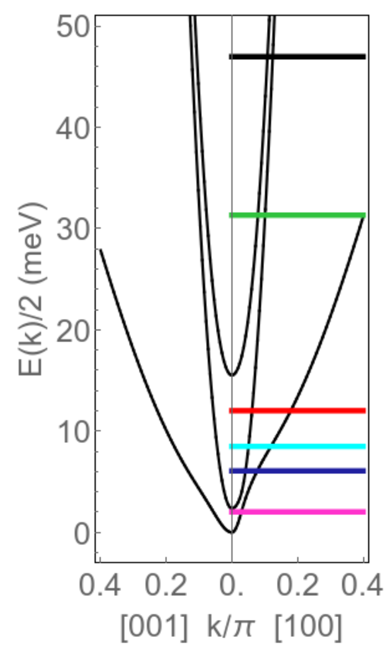


Theory





McCalla, MNG, Cassuto, Fernandes, Leighton, in prep.

Van Der Marel et al., PRB (11)

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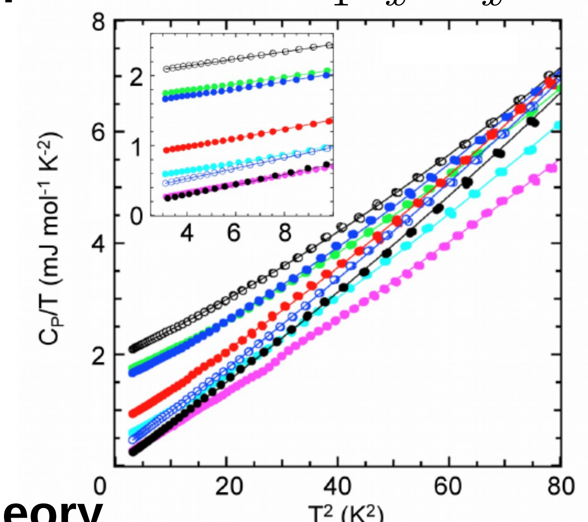

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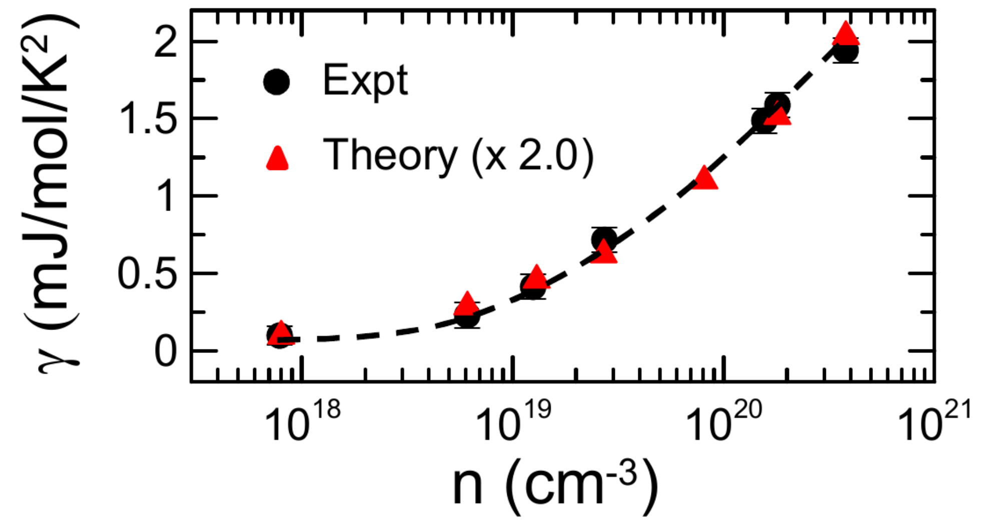

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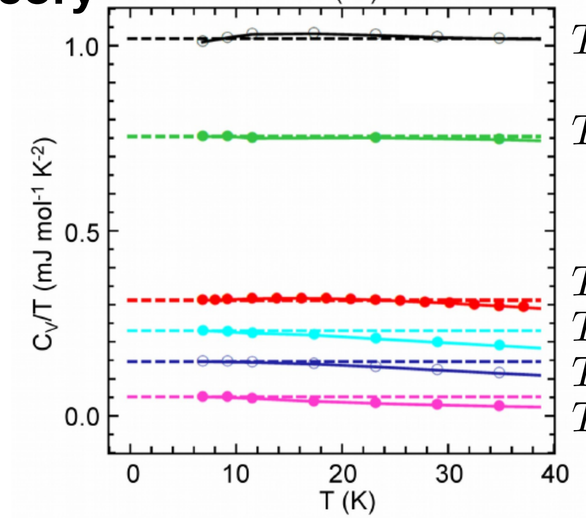


$T_F = 540\text{K}$

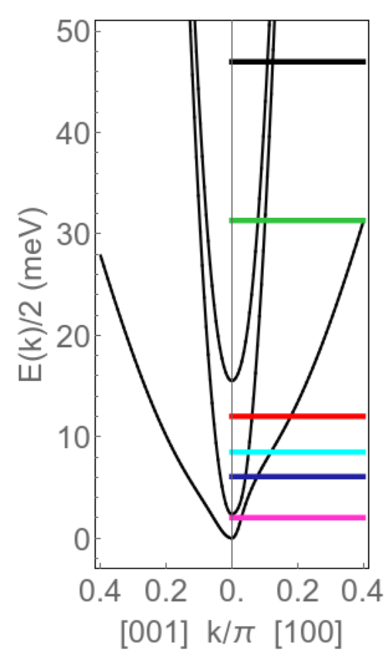
$T_F = 360\text{K}$

$T_F = 140\text{K}$

$T_F = 100\text{K}$

$T_F = 70\text{K}$

$T_F = 23\text{K}$



McCalla, MNG, Cassuto, Fernandes, Leighton, in prep.

Experimental values of γ reproduced with:

1) moderate mass enhancement ***factor 2***

2) ***independent*** of n

$$\frac{\gamma_{\text{exp}}}{\gamma_{\text{band}}} \sim \frac{m^*}{m} \sim 2$$

REPORTS

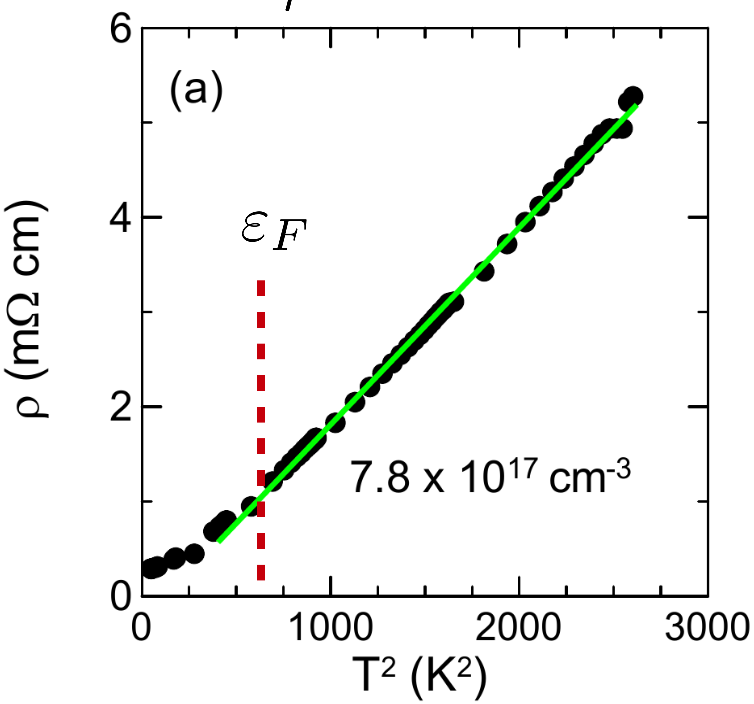
SOLID-STATE PHYSICS

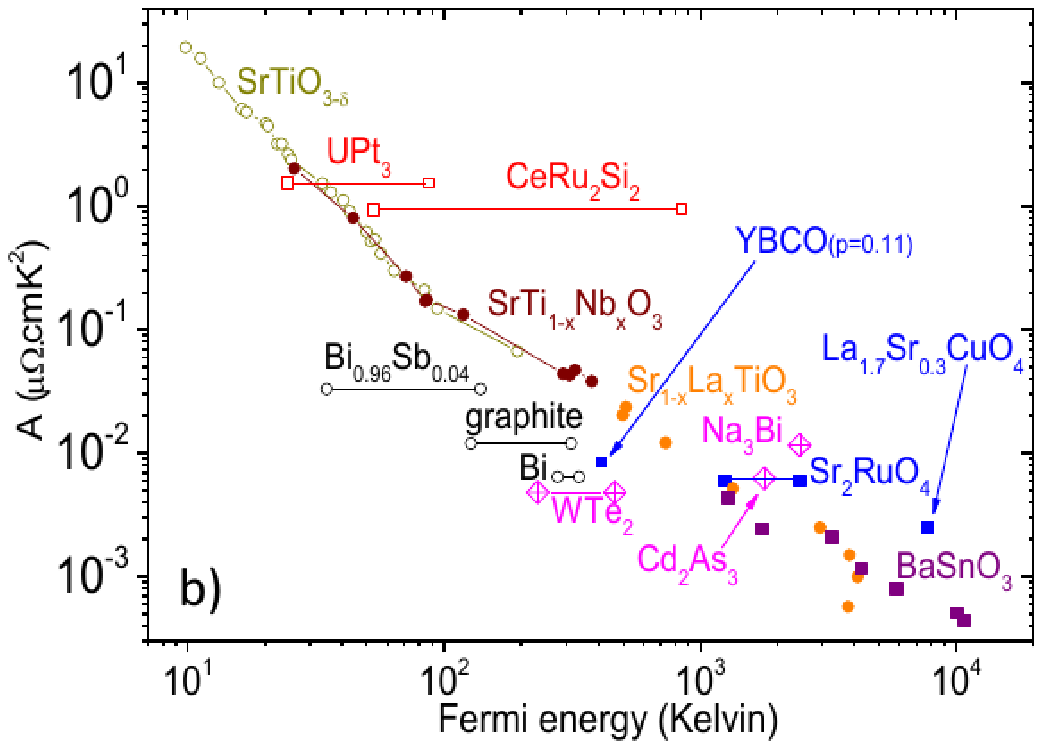
Scalable T^2 resistivity in a small single-component Fermi surface

Xiao Lin, Benoît Fauqué, Kamran Behnia*

Mechanism for T-square resistivity?

$$\rho \sim AT^2$$





Kadowaki-Woods scaling:

$$\left.\begin{aligned} C \sim \gamma T \longrightarrow \gamma \sim m^* \\ \rho \sim AT^2 \longrightarrow A \sim (m^*)^2 \end{aligned}\right\} \boxed{\frac{A}{\gamma^2} = R_{KW}}$$

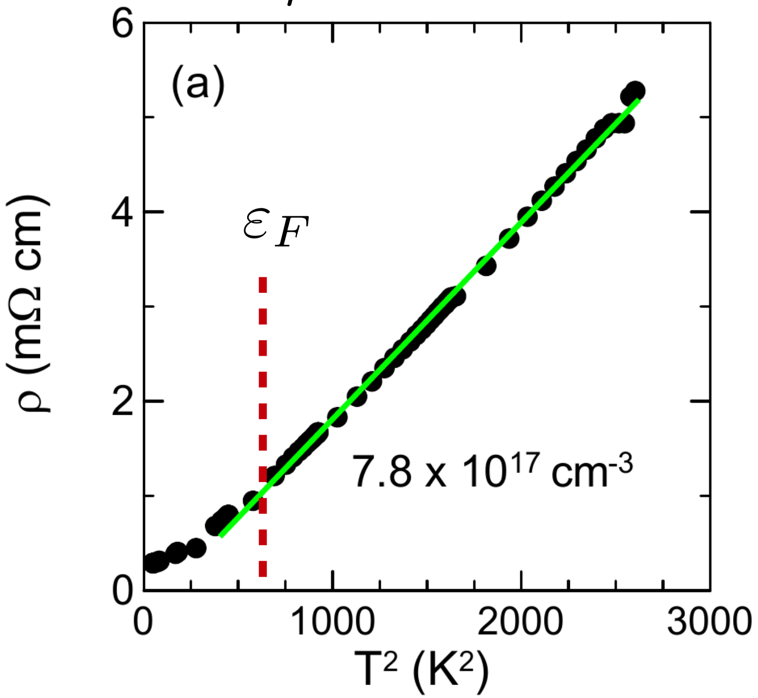
McCalla, MNG, Cassuto, Fernandes, Leighton, in prep.

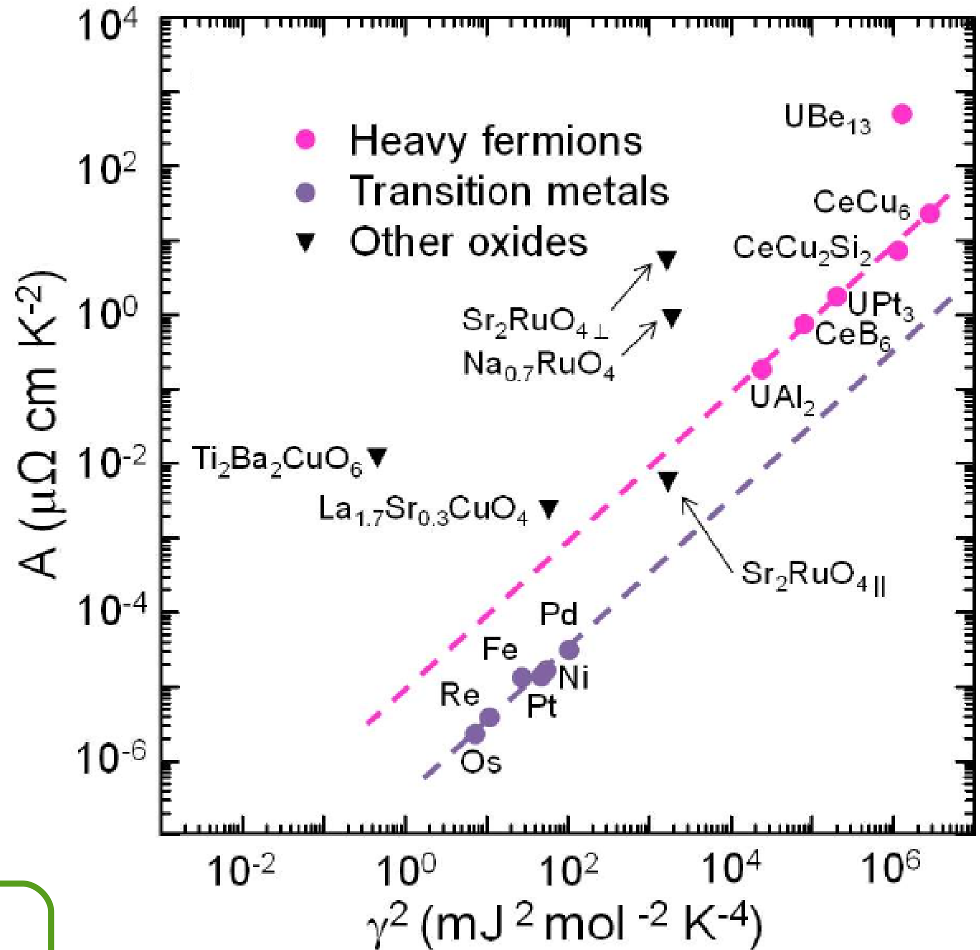
'A scales inversely with the Fermi energy, an extension of the Kadowaki-Woods scaling in low-density metals'

Lin et al, Science (15)
Collignon et al, arXiv (18)

Mechanism for T-square resistivity?

$$\rho \sim AT^2$$





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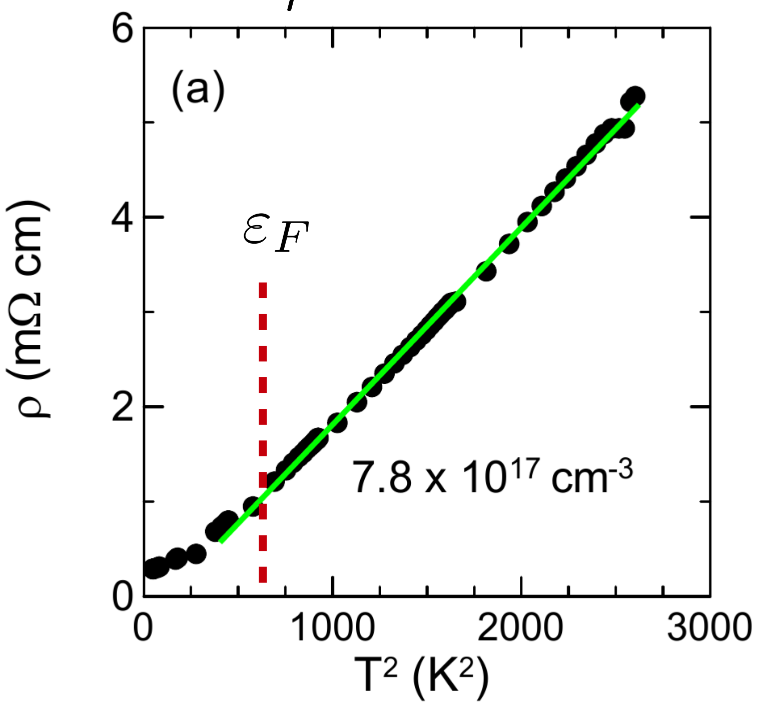

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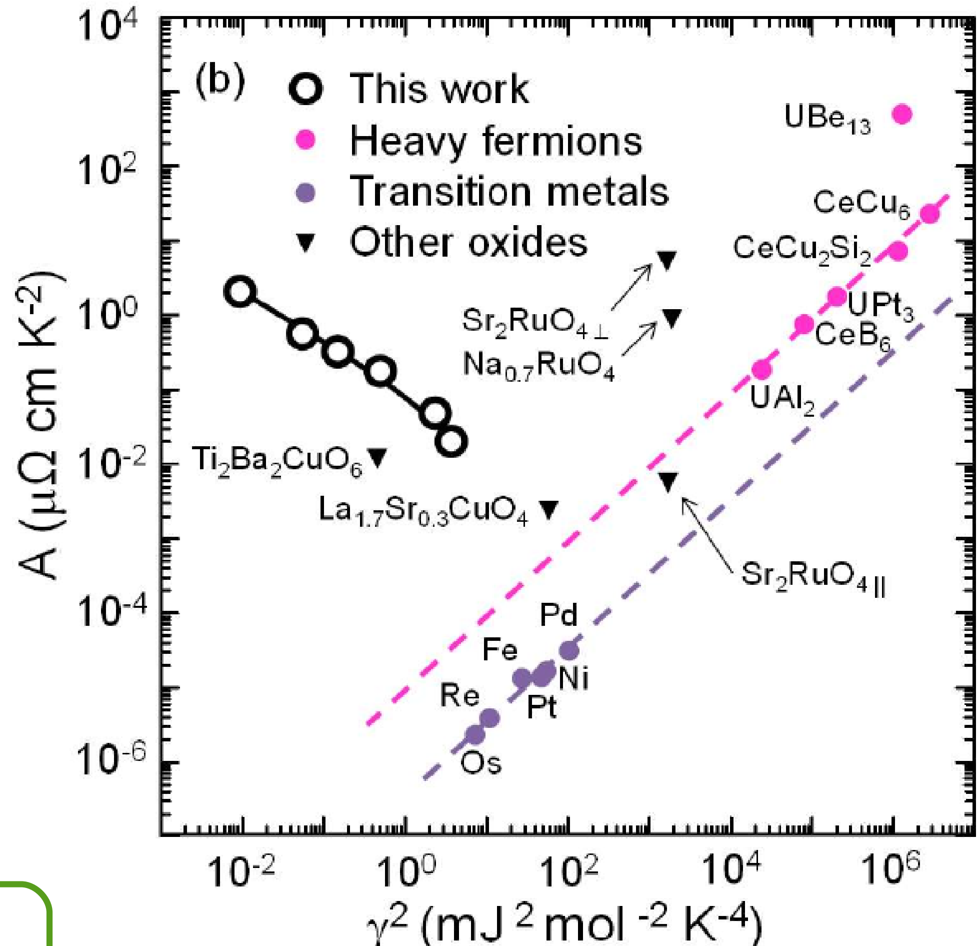


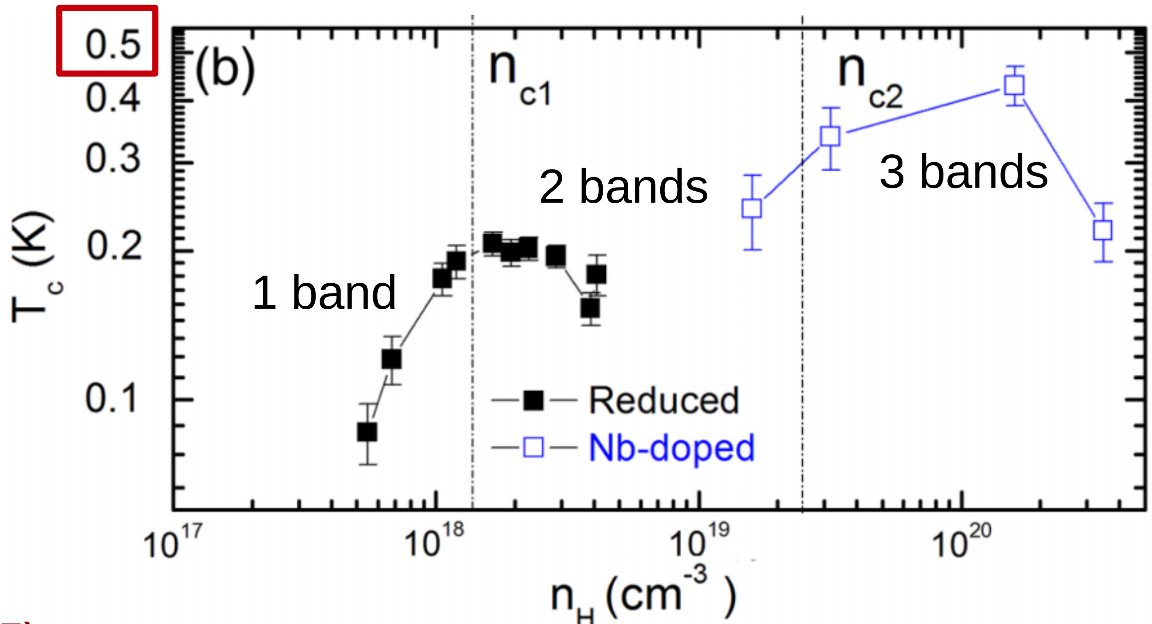
Opposite trend!!

See also:

Swift & Van de Walle, Eur. Phys. J. B (17)

Hussey, J. Phys. Soc. Japan (05)

Superconducting state



2 meV

40 meV

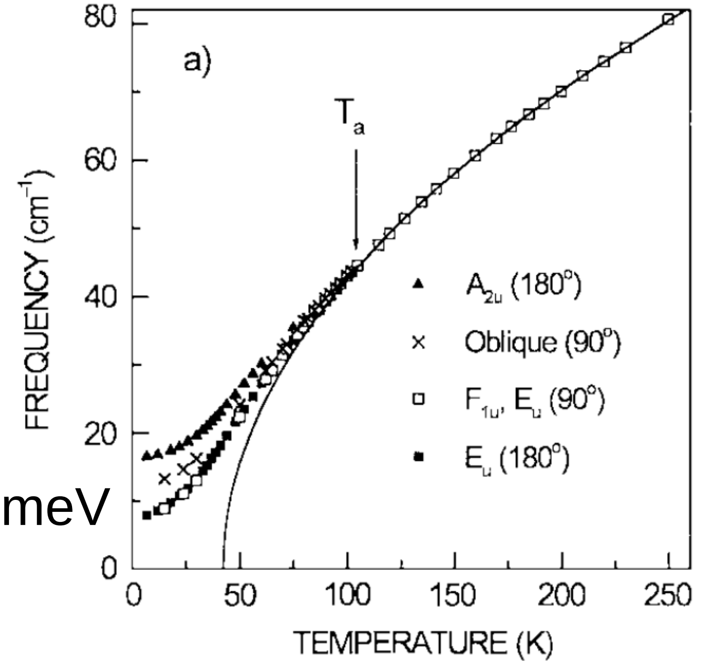
See also:

Edge et al., PRL (15)
Gor'kov, PNAS (16)
Ruhman et al., PRB (16)
Rowley et al., arXiv (18)

Polar phonons in SrTiO

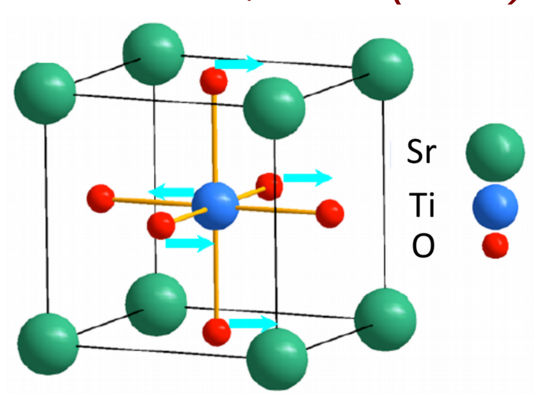
INCIPIENT FERROELECTRIC

Transverse Optical soft mode



1 meV

Yamanaka et al., EPL (2000)



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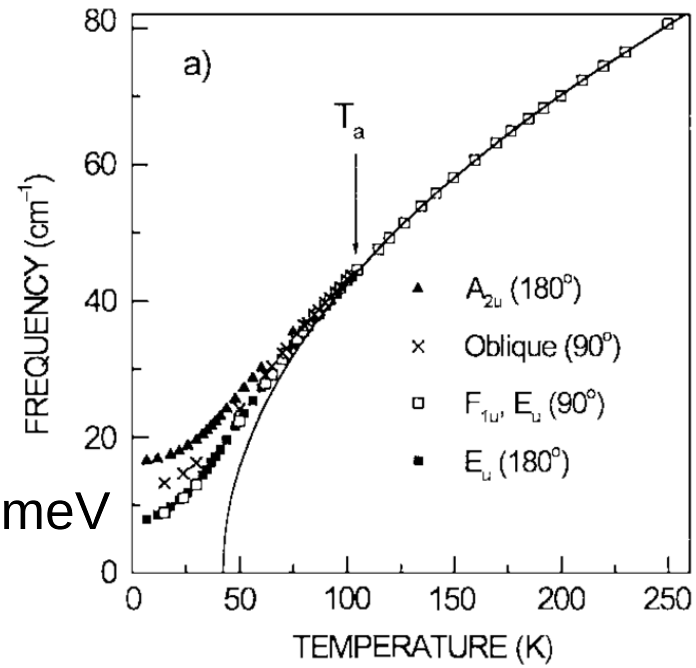

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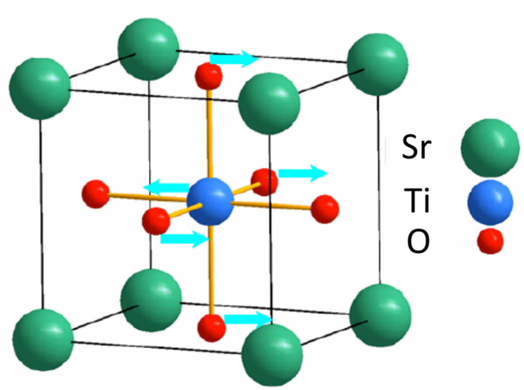
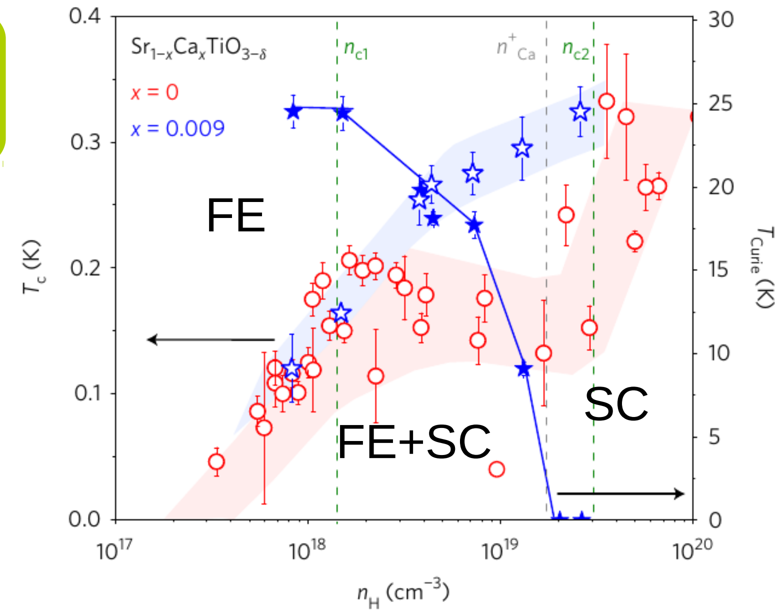


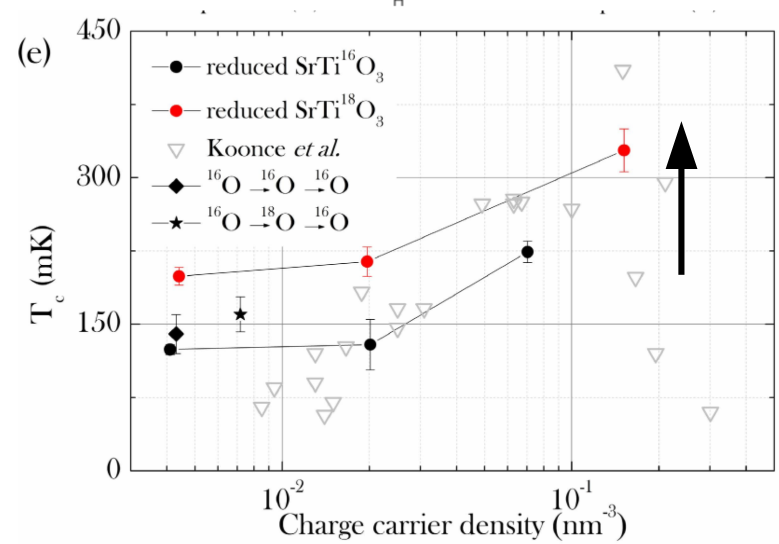
Enhanced SC close to FE

Rischau et al., Nat. Phys (17)

$x(^{18}O)/x(^{16}O)=0.35$

Stucky et al., Sci. Rep (16)





Electron-phonon coupling

Experimental indication for relevance of OPTICAL PHONONS

Electron-phonon coupling

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$$\mathcal{H}_{el-ph} = \sum_{\mathbf{k},\mathbf{q}} g(\mathbf{q}) \hat{c}^{\dagger}_{\mathbf{k}+\mathbf{q}} \hat{c}_{\mathbf{k}} (\hat{a}^{\dagger}_{-\mathbf{q}} + \hat{a}_{\mathbf{q}})$$

where

$$g(\mathbf{q}) \propto \frac{1}{\sqrt{M\omega_{\mathbf{q}}}} \mathbf{q} \cdot \hat{\mathbf{e}}(\mathbf{q}) V_0$$

$$\begin{cases} \mathbf{q} \perp \hat{\mathbf{e}} \longrightarrow & g_{TO} \ll 1 \\ \mathbf{q} \parallel \hat{\mathbf{e}} \longrightarrow & g_{LO} \gg g_{TO} \end{cases}$$

$$\boxed{\Omega_{LO} >> \varepsilon_F}$$

Unconventional limit for e-ph SC

Effective potential experienced by the electrons. Depends on the type of phonon

Bardeen-Pines interaction

$$\tilde{V} = \frac{V(q)}{\varepsilon_{el,i}(q,\Omega_n)} = \frac{e^2}{q^2+\kappa^2}\left[1 - \frac{\Omega_L^2}{\Omega_n^2+\Omega_L^2}\right]$$

Screening from Fermi sea and ionic plasma

Linearized gap-equation for a 3D parabolic dispersion:

$$\phi(\epsilon,\omega_n) = \sqrt{\mathrm{Ry}}\frac{T}{\pi}\sum_{n'}\left[\frac{1}{1+|\omega_n'-\omega_n|^2} - 1\right]\int_0^\Lambda d\epsilon' \frac{\sqrt{\epsilon'}}{{\omega_n'}^2+(\epsilon'-\mu)^2} v(\epsilon,\epsilon',\kappa^2)\phi(\epsilon',\omega_n')$$

with the singular interaction

$$v(\epsilon,\epsilon',\kappa^2) = \frac{1}{2\sqrt{\epsilon\epsilon'}}\log\left[\frac{(\sqrt{\epsilon}+\sqrt{\epsilon'})^2+\kappa^2}{(\sqrt{\epsilon}-\sqrt{\epsilon'})^2+\kappa^2}\right]$$

The Thomas-Fermi screening is a function of the chemical potential

$$\kappa^2 = \frac{4}{\pi}\sqrt{\mu \mathrm{Ry}}$$

Rydberg unit of energy Ry=13.6 eV

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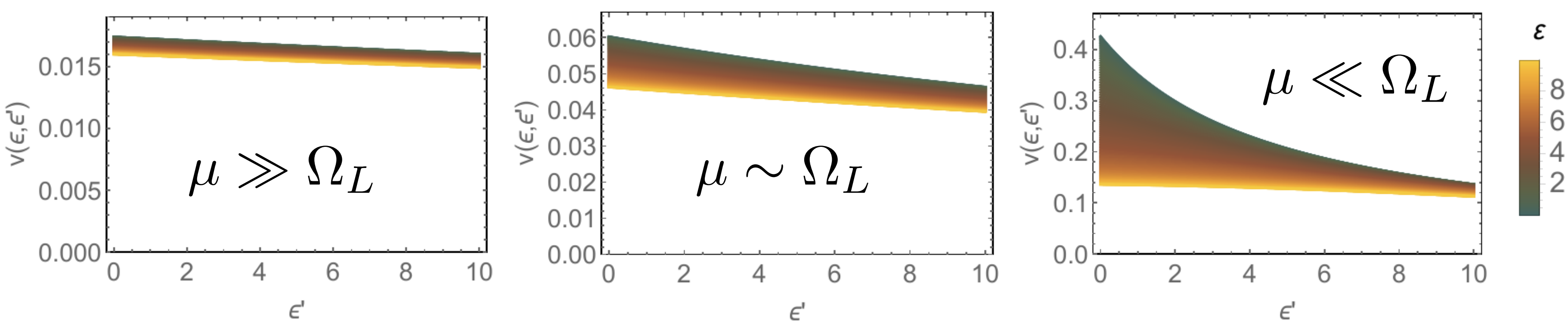

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$\boxed{\textbf{BCS approx}}$: (I) neglect repulsion (Morel & Anderson)

(II) $\mu \gg \Omega_L$ constant interaction and DOS

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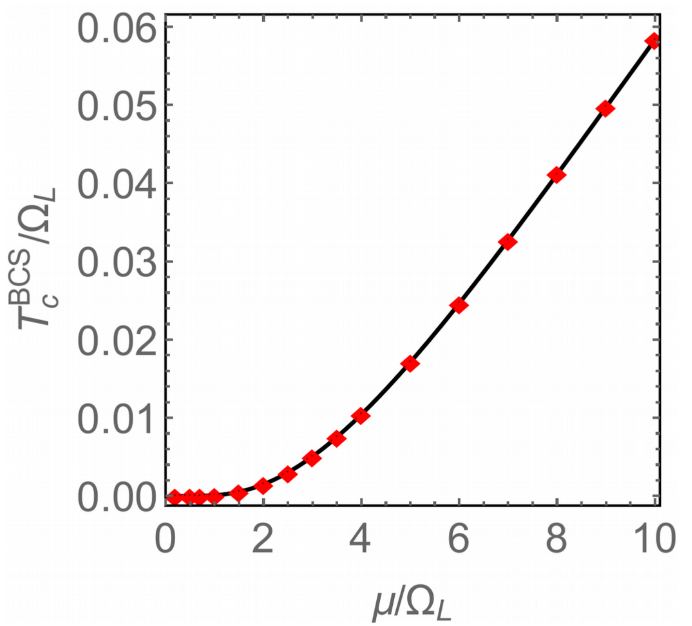
$$1 = \frac{D(\mu)V_0}{2}T\sum_{n'}\int_{-\Omega_L}^{\Omega_L} d\xi' \frac{1}{{\omega_n'}^2+\xi'^2}$$

$$\boxed{\frac{T_c^{BCS}}{\Omega_L} \sim 1.13 e^{-\frac{1}{D(\mu)V/2}}}$$

Fast decrease of Tc when decreasing E_F



Bardeen-Pines interaction

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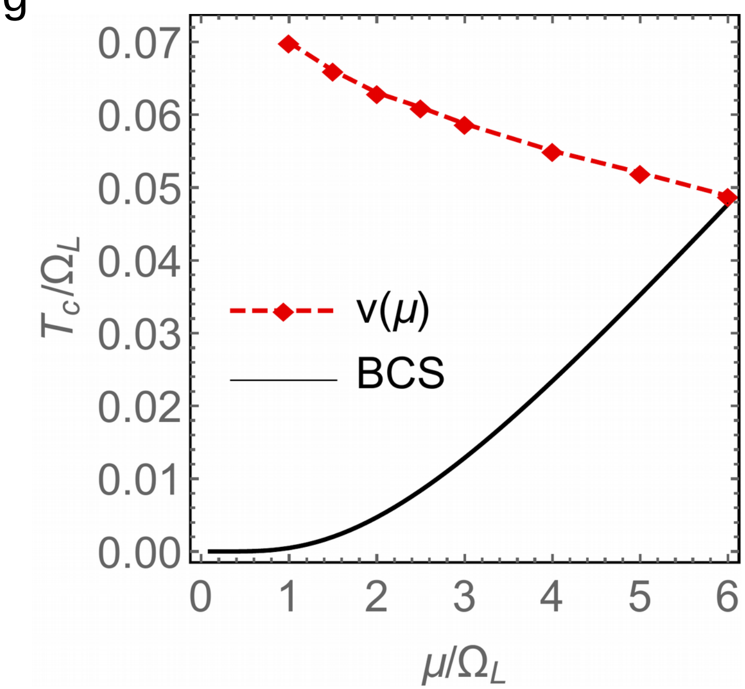

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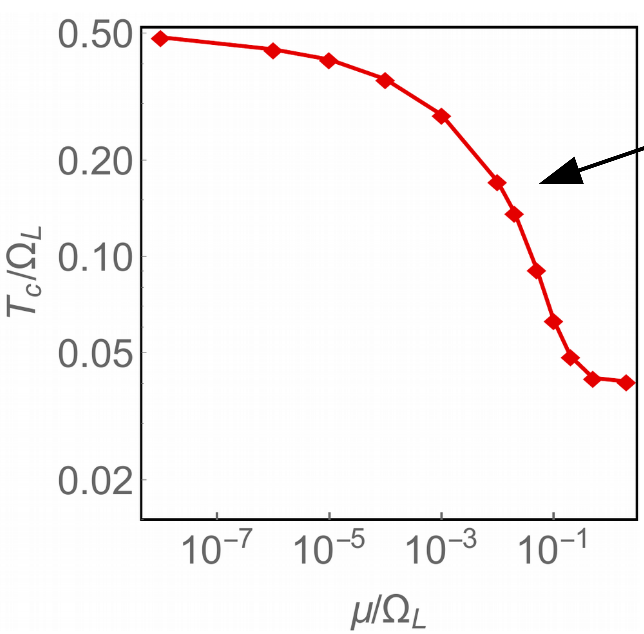
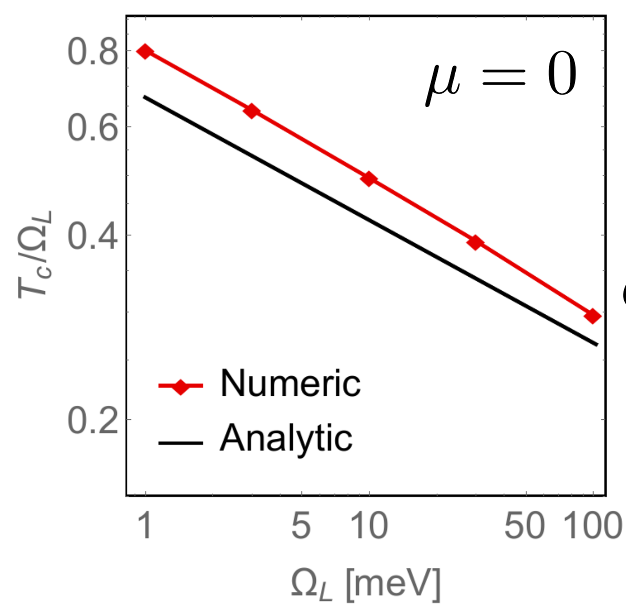


Increase of Tc when decreasing E_F (opposite trend of BCS limit)



$$\boxed{T_c = C(\mathrm{Ry}/\Omega_L)^{\frac{1}{5}}}$$

$$C = \frac{1}{\pi^{\frac{9}{5}}(5-\sqrt{10})^{\frac{2}{5}}}$$

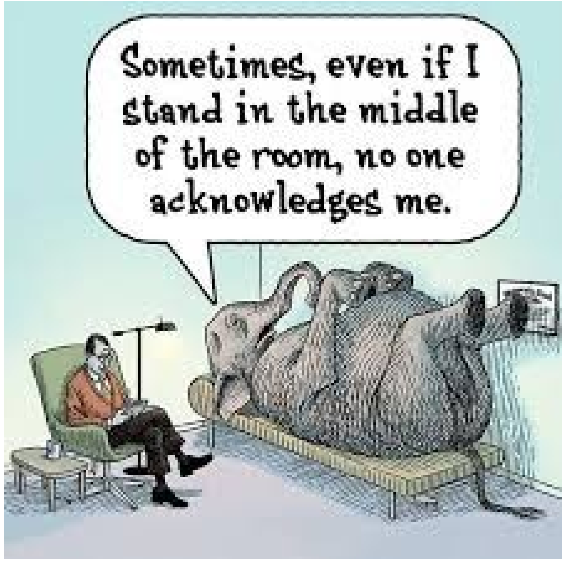
Outlook: Coulomb repulsion

Linearized gap-equation:

$$\phi(\tilde{\epsilon},\tilde{\omega}_n) = \sqrt{\tilde{R}y}\frac{\tilde{T}}{\pi}\sum_{n'\neq n}\left[\frac{1}{1+|\tilde{\omega}'_n-\tilde{\omega}_n|^2} - 1\right]\int_0^{\tilde{\Lambda}} d\tilde{\epsilon}'\,\frac{\sqrt{\tilde{\epsilon}'}}{\tilde{\omega}'^{\,2}_n+(\tilde{\epsilon}'-\tilde{\mu})^2}v(\tilde{\epsilon},\tilde{\epsilon}',\tilde{\kappa}^2)\phi(\tilde{\epsilon}',\tilde{\omega}'_n)$$

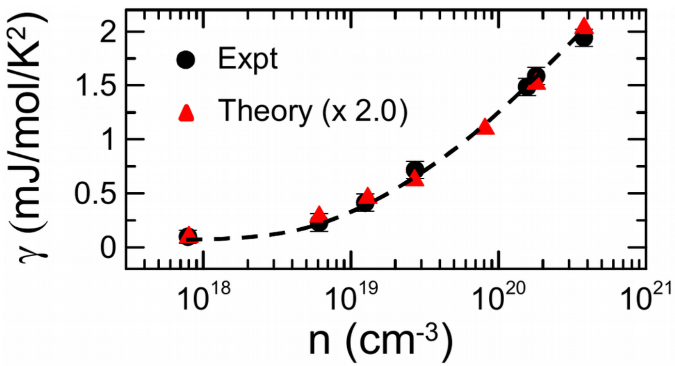
Pseudopotential mechanism: electron-phonon attraction may overcome Coulomb repulsion.

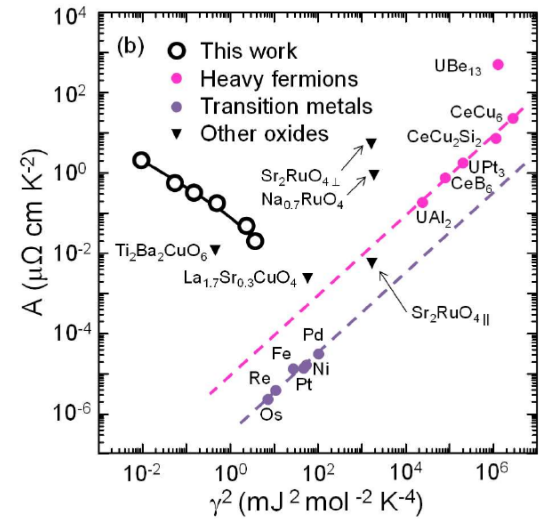
Does it hold in the $\Omega_{LO} >> \varepsilon_F$ limit?



Conclusions

- The Normal state
  - Weakly correlated Fermi liquid
  - Unusual mechanism for T^2 resistivity?





- Superconducting state by Bardeen-Pines interaction
  - 'Non-BCS' behavior in the limit of $\varepsilon_F << \Omega_L$

    Tc increases with decreasing ε_F, and saturates.
  - Does SC survive the inclusion of repulsion?

$$T_c = C(\mathrm{Ry}/\Omega_L)^{\frac{1}{5}}$$